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Reggeized differential cross sections for fermion-exchange processes in the covariant formalism

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Abstract. A scheme developed by Scadron and Gault for calculating reggeized spin-averaged differential cross sections for boson exchange is extended, via s channel helicity vertices, to backward processes involving fermion Regge pole exchanges including those initiated by photons.

Using the higher spin propagator formalism of Scadron (1968), a method of reggeizing invariant amplitudes has been developed by Jones and Scadron (1968a, 1968b), Gault (1969) and Gault *et al.* (1970). Within this formalism, Gault and Scadron (1970) were able to develop a scheme for calculating reggeized spin-averaged differential cross sections for boson-exchange processes.

In this paper we perform similar but more complicated calculations for processes involving fermion Regge poles, where factorization is not automatic, as for the boson case†, and Gribov parity doubling of fermion trajectories is also involved (Gribov 1963, Gribov *et al.* 1964). However, the residue functions in each of the s channel helicity amplitudes do factorize asymptotically; this makes it possible to construct many cross sections from a few vertex traces, which are calculated and listed in table 2. Backward photonic processes are also considered, incorporating gauge invariance (Appendix 1).

In the covariant formalism developed by Jones, Scadron and Gaunt, the partial wave expansion in the u channel is reggeized by the prescription

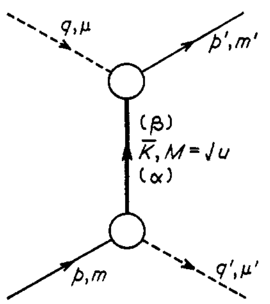
$$c_J \mathscr{P}_J(z_u) \rightarrow (\tau)^{\alpha(\sqrt{u})} \xi_{\pm}(\sqrt{u}) \qquad (1)$$

$$J \rightarrow \alpha(\sqrt{u})$$

coupling constants $\rightarrow$ Regge residue functions of $\sqrt{u}$, where $\xi_{\pm}$ is the signature factor and

$$\tau(\bar{K}) = \tau - \frac{\Lambda'.\bar{K}\Lambda.\bar{K}}{u} \sim \tau = \Lambda'.\Lambda = \tfrac{1}{4}(s-t).$$

With the kinematics defined in figure 1 (Scadron 1968)



$$\Lambda' = \tfrac{1}{2}(p'+q)$$
$$\Lambda = \tfrac{1}{2}(p+q')$$
$$\bar{K} = (p-q') = (p'-q)$$
$$J = j+\tfrac{1}{2} \qquad (j \text{ integral})$$

Figure 1.

† For references see Gell-Mann (1962), Gribov and Pomeranchuck (1962) and Gault and Scadron (1970).

the spin averaged differential cross section in the asymptotic region is given by

$$\tau^2 \frac{\mathrm{d}\sigma}{\mathrm{d}u} \sim \sum_{(\lambda)} |T_{(\lambda)}|^2 = \mathrm{Tr}(\mathscr{M}_{fi}\mathscr{P}_{ii'}\bar{\mathscr{M}}_{f'i'}\mathscr{P}_{f'f}) \tag{2}$$

where

$$\mathscr{M}_{fi} = \mathscr{C}_{f\beta}\mathscr{S}_{\beta;\alpha}{}^{J}(\Lambda', \Lambda; \bar{K})\mathscr{C}_{i\alpha}.$$

Here $\mathscr{S}_{\beta;\alpha}{}^{J}$ is the contracted spin J fermion propagator numerator, $\mathscr{P}_{ii'}$ and $\mathscr{P}_{ff'}$ are the products of the projection operators for the initial and final particles in the u channel, and the $\mathscr{C}$ are the reduced Regge couplings displayed in table 1.

Table 1. Reduced Regge couplings (BFF)

$\mathscr{C}^{+}(0, \frac{1}{2}, j+\frac{1}{2})$	(g)
$\mathscr{C}_{\alpha\nu}{}^{+}(1, \frac{1}{2}, j+\frac{1}{2})$	$(g_1 g_{\alpha\nu} + g_2\Lambda_\alpha\gamma_\nu + g_3\Lambda_\alpha\Lambda_\nu)$
$\mathscr{C}_{\alpha\mu}{}^{+}(0, \frac{3}{2}, j+\frac{1}{2})$	$(g_1 g_{\alpha\mu} + g_2\Lambda_\alpha\Lambda_\mu)$
$\mathscr{C}^{+}_{\alpha_1\alpha_2\mu\nu}(1, \frac{3}{2}, j+\frac{1}{2})$	$(g_1 g_{\alpha_1\mu}g_{\alpha_2\nu} + g_2 g_{\alpha_1\mu}\gamma_\nu\Lambda_{\alpha_2} + g_3 g_{\alpha_1\mu}\Lambda_{\alpha_2}\Lambda_\nu$ $+ g_4 g_{\mu\nu}\Lambda_{\alpha_1}\Lambda_{\alpha_2} + g_5\gamma_\nu\Lambda_{\alpha_1}\Lambda_{\alpha_2}\Lambda_\mu + g_6\Lambda_{\alpha_1}\Lambda_{\alpha_2}\Lambda_\mu\Lambda_\nu)$
abnormal couplings:	$g \to \gamma_5 f$

(see Appendix 1 for photonic couplings).

As the partial-wave amplitudes in the u channel satisfy the McDowell (1969) symmetry rule

$$T^{J}_{J-1/2}(\sqrt{u}) = -T^{J}_{J+1/2}(-\sqrt{u}) \tag{3}$$

where the subscripts stand for $l = J \pm \frac{1}{2}$, the Regge pole contribution of the Gribov parity partner must also be included in the $\mathscr{M}$ function. Therefore we write

$$\mathscr{M} = \mathscr{M}^{+} + \mathscr{M}^{-} \tag{4}$$

where $\mathscr{M}^{+}$ and $\mathscr{M}^{-}$ are the $\mathscr{M}$ functions for the exchange of the normal parity Regge pole and its parity partner, the abnormal parity Regge pole, respectively. Then the cross section becomes

$$\tau^2 \frac{\mathrm{d}\sigma}{\mathrm{d}u} \sim \mathrm{Tr}\{(\mathscr{M}^{+} + \mathscr{M}^{-})_{fi}\mathscr{P}_{ii'}(\overline{\mathscr{M}^{+} + \mathscr{M}^{-}})_{f'i'}\mathscr{P}_{f'f}\}$$
$$= I^{++} + I^{--} + I^{+-} + I^{-+} \tag{5}$$

where

$$I^{++} = \mathrm{Tr}(\mathscr{M}_{fi}{}^{+}\mathscr{P}_{ii'}\bar{\mathscr{M}}_{f'i'}{}^{+}\mathscr{P}_{f'f})$$

etc, in which the term I^{--} can be obtained using equation (3). At this point one should note that the trace does not break up into two parts as in the case of boson-exchange processes (Gault and Scadrón 1970), and thus factorization is not automatically satisfied. But in the following pages we will show that factorization is in fact satisfied *in asymptopia*.

Now, it can easily be shown that the terms containing $\mathscr{M}$ functions of the same normality, that is, I^{++} and I^{--}, contribute to leading order, whereas the interference terms I^{+-} and I^{-+}, contributing one order lower than the leading order, may be neglected in the asymptotic region.

Since direct trace calculations are rather clumsy and become extremely tedious for higher-spin processes, we instead write the T matrix in s channel helicity form (Jacob and Wick 1959, Gault and Jones 1971, see Appendix 2) and use the cross section formula

$$\tau^2 \frac{d\sigma}{du} \sim \sum_{all\,\lambda} |T^J_{\lambda_4\lambda_3;\lambda_2\lambda_1}|^2 \tag{11}$$

where

$$T^J_{\lambda_4\lambda_3;\lambda_2\lambda_1} \sim \bar{u}_\mu{}'^{(\lambda_3)}(p')\epsilon_\nu{}^{(\lambda_2)}(q)\mathscr{C}_{\mu\nu\beta}\mathscr{S}_{\beta;\alpha}{}^J\mathscr{C}_{\alpha\mu'\nu'}\epsilon_{\nu'}{}'^{(\lambda_4)*}(q')u_{\mu'}{}^{(\lambda_1)}(p) \tag{12}$$

where λ_1, λ_2, λ_3 and λ_4 are the s helicity labels and μ, μ' and ν, ν', the spinor and vector labels of the respective particles.

In the following we give a few examples to clarify the present technique and to show explicit factorization of the residue functions of the amplitudes *in asymptopia*:

(i) $\pi N \rightarrow N\pi$ (Singh 1963, Chiu and Stack 1967, Gribov 1963 and Gribov *et al.* 1964).

For normal parity exchange equation (12) gives

$$T^J_{\lambda';\lambda} \sim \bar{u}^{(\lambda')}(p')(\gamma_5 f)\mathscr{S}^J(r_5 f)u^{(\lambda)}(p)$$

$$= \frac{c_{j+1}}{j+1}(f)^2\bar{u}^{(\lambda')}(p')\gamma_5\left(\frac{(\gamma.\bar{K}+M)}{2M}\mathscr{P}'_{j+1} - \gamma.\Lambda'(\bar{K})\frac{(\gamma.\bar{K}-M)}{2M}\gamma.\Lambda(\bar{K})\mathscr{P}_j{}'\right)\gamma_5 u^{(\lambda)}(p). \tag{13}$$

The first term in equation (13) clearly contributes to leading order, whereas the second term gives a lower order contribution†, and can be ignored asymptotically. This argument can also be shown to be true for more complicated fermion propagators which arise in higher spin processes.

Therefore equation (13) becomes

$$T^J_{\lambda';\lambda} \sim (f)^{2\prime}\bar{u}^{(\lambda')}(p')\frac{(\gamma.\bar{K}-M)}{2M}u^{(\lambda)}(p)\left(\frac{c_{j+1}}{j+1}\mathscr{P}_{j+1}{}'\right)$$

which gives‡ (Appendix 2)

$$T^J_{1/2;1/2} = T^J_{-1/2;-1/2} \sim -(f)^2\frac{\sqrt{-u}}{4mM}\tau^J \tag{14}$$

$$T^J_{1/2;-1/2} = -T^J_{-1/2;1/2} \sim -(f)^2\frac{1}{4m}\tau^J$$

and thus

$$I^{++} = \sum_{\lambda',\lambda}|T^J_{\lambda';\lambda}|^2 \sim \frac{|f|^4}{4m^2}\tau^{2J}. \tag{15}$$

In this case the residue function obviously factorizes since the π–N reggeon coupling is just $(r_5 f)$ for normal and (g) for abnormal exchanges.

(ii) $\pi N \rightarrow N\rho_\mu$ (Appendix 2): For abnormal parity exchange equation (12) gives

$$T^J_{\lambda''\lambda';\lambda} \sim \bar{u}^{(\lambda')}(p')(\gamma_5 f)\mathscr{S}_{;\alpha}{}^J(g_1 g_{\alpha\mu} + g_2\Lambda_\alpha\gamma_\mu + g_3\Lambda_\alpha\Lambda_\mu)\epsilon_\mu{}'^{*(\lambda'')}(q')u^{(\lambda)}(p)$$

† First term $= \bar{u}\frac{(\gamma.\bar{K}-M)}{2M}u\frac{c_{j+1}}{j+1}\mathscr{P}_{j+1}{}' \sim \bar{u}\frac{(\gamma.\bar{K}-M)}{2M}u(\tau^j)$

second term $= \bar{u}\frac{(\gamma.\bar{K}+M)}{2M}u\left(m - \frac{P'.\bar{K}}{M}\right)\left(m - \frac{P.\bar{K}}{M}\right)\frac{c_{j+1}}{j+1}\mathscr{P}_j{}' \sim \bar{u}\frac{(\gamma.\bar{K}+M)}{2M}u(\tau^{j-1})$.

‡ Our choice of normalization is $\sum_\lambda u^{(\lambda)}(p)\bar{u}^{(\lambda)}(p) = \frac{(\gamma.p+m)}{2m}$.

which, to leading order, can be written as

$$\sim \bar{u}^{(\lambda')}(p')(\gamma_5 f)\frac{(\gamma.\bar{K}+M)}{2M}(g_1\epsilon_\alpha{}'^{*(\lambda'')}(q')+g_2\Lambda_\alpha\gamma.\epsilon'^{*(\lambda'')}(q')+g_3{}'\Lambda_\alpha\Lambda.\epsilon'^{*(\lambda'')}(q'))$$

$$\times u^{(\lambda)}(p)\left(\frac{c_{j+1}}{j(j+1)}\mathscr{P}'_{;\alpha}{}^{(j+1)}\right). \tag{16}$$

We now show that each term in the parenthesis of equation (16) decouples, asymptotically and near the backward direction, from the fermion trace part, that is, the term $\bar{u}\gamma_5\,(\gamma.\bar{K}+M)\,u$, and the residue function factorizes for each helicity amplitude.

Since no γ matrices are involved in the first and the third terms of the parenthesis, they clearly decouple from the fermion trace part for each explicit helicity amplitude. Thus, for these two terms, we only have to calculate the contributions of the fermion trace part which are essentially the same as in the πN case before. The only difference is the sign factor coming from the presence of one γ_5 between the spinors.

The contributions are

$$\bar{u}^{(1/2)}(p')\gamma_5\frac{(\gamma.\bar{K}+M)}{2M}u^{(1/2)}(p) \sim -\mathrm{i}\frac{\sqrt{-u}}{4mM}\tau^{1/2}$$

$$\bar{u}^{(1/2)}(p')\gamma_5\frac{(\gamma.\bar{K}+M)}{2M}u^{(-1/2)}(p) \sim \frac{\mathrm{i}}{4m}\tau^{1/2} \tag{17}$$

which clearly gives the relation

$$\bar{u}^{(1/2)}(p')\gamma_5\frac{(\gamma.\bar{K}+M)}{2M}u^{(1/2)}(p) = \mathrm{i}\bar{u}^{(1/2)}(p')\gamma_5\frac{(\gamma.\bar{K}+M)}{2M}u^{(-1/2)}(p). \tag{18}$$

Now for the second term, consider the part $\gamma.\,\epsilon'^{*(\lambda'')}(q')u^{(\lambda)}(p)$ for each helicity state of the polarization vector. Then we find in the asymptotic region that (Appendix2)

$$\gamma.\epsilon'^{*(0)}(q')u^{(\lambda)}(p) = \frac{\gamma.q'}{\mu'}u^{(\lambda)}(p)$$

$$= \frac{(m-M)}{\mu'}u^{(\lambda)}(p) \tag{19†}$$

while

$$\gamma.\epsilon'^{*(\pm)}(q')u^{(\lambda)}(p) = \gamma.\epsilon'^{*(\pm)}(q')(E+m)^{1/2}\left(1+\frac{\mathrm{i}\gamma_5 ph}{E+m}\right)\phi^{(\lambda)}(\hat{\boldsymbol{p}})$$

$$\sim \pm h\left(\frac{E}{2}\right)^{1/2}(1-\mathrm{i}\gamma_5 h)\sigma_\mp\,\phi^{(\lambda)}(\hat{\boldsymbol{p}}). \tag{20†}$$

† We use the relations: $(\gamma.\bar{K}+M)\gamma.\bar{K} = (\gamma.\bar{K}+M)M$ and $(\gamma.p-m)u(p) = 0$ for equation (19) and the definition $\sigma_\pm = \sigma_1 \pm \mathrm{i}\sigma_2$ for equation (20).

Thus we have

$$
\begin{aligned}
\gamma.\epsilon'^{*(+)}(q')u^{(1/2)}(p) &\sim \sqrt{2}u^{(-1/2)}(p) = \mathrm{i}\sqrt{2}u^{(1/2)}(p)\\
\gamma.\epsilon'^{*(-)}(q')u^{(1/2)}(p) &\sim 0\\
\gamma.\epsilon'^{*(+)}(q')u^{(-1/2)}(p) &\sim 0\\
\gamma.\epsilon'^{*(-)}(q')u^{(-1/2)}(p) &\sim \sqrt{2}u^{(1/2)}(p) = \mathrm{i}\sqrt{2}u^{(-1/2)}(p)
\end{aligned}
\tag{21}
$$

where equation (18) has been used.

Equations (19) and (21) clearly show that not only the first and the third but also the second term in equation (16) decouples from the fermion trace part and thus exhibits factorization of the Regge residue for each helicity amplitude.

From equations (16), (17), (19) and (21), the final results can be written as

$$
\begin{aligned}
T^J_{0,1/2;1/2} &= T^J_{0,-1/2;-1/2} = \mathrm{i}T^J_{0,1/2;-1/2} = \mathrm{i}T^J_{0,-1/2;1/2}\\
&\sim -\frac{(f)}{4m\mu'}\{g_1+(m-\sqrt{u})g_2+\tfrac{1}{4}(m^2-\mu'^2-u)g_3\}\tau^J\\
T^J_{1,1/2;1/2} &= T^J_{-1,-1/2;-1/2} = -\mathrm{i}T^J_{-1,1/2;-1/2} = -\mathrm{i}T^J_{1,-1/2;1/2}\\
&\sim \frac{(f)}{4m}\left(\mathrm{i}\sqrt{2}g_2+\frac{\sqrt{-u}}{2\sqrt{2}}g_3\right)\tau^J\\
T^J_{-1,1/2;1/2} &= T^J_{1,-1/2;-1/2} = -\mathrm{i}T^J_{1,1/2;-1/2} = -\mathrm{i}T^J_{-1,-1/2;1/2}\\
&\sim -\frac{(f)}{4m}\left(\frac{\sqrt{-u}}{2\sqrt{2}}g_3\right)\tau^J
\end{aligned}
\tag{22}
$$

and thus the contribution

$$
\begin{aligned}
I^{++} \sim \frac{|f|^2}{4m^2}\Bigg(&\frac{1}{\mu'^2}\,|g_1+(m-\sqrt{u})g_2+\tfrac{1}{4}(m^2-\mu'^2-u)|^2\\
&+\left|\mathrm{i}\sqrt{2}g_2+\frac{\sqrt{-u}}{2\sqrt{2}}g_3\right|^2+\left|\frac{\sqrt{-u}}{2\sqrt{2}}g_3\right|^2\Bigg)
\end{aligned}
\tag{23}
$$

which can be checked by direct trace calculation.

From the above two examples, it is clear that in general one could write down equation (12), asymptotically, in the form

$$
T_{\{\lambda\}}{}^J \sim \{A\}\{B\}\tau^J \tag{24}
$$

where $\{A\}$ and $\{B\}$ are the decoupled Regge residue functions or the helicity vertex functions for each vertex (Jacob and Wick 1959, Gault and Jones 1971).

The contribution to the cross section is then

$$
\begin{aligned}
|T_{\{\lambda\}}|^2 &\sim \{A\}^2\{B\}^2\tau^{2J}\\
&= [A][B]\tau^{2J}.
\end{aligned}
\tag{25}
$$

Thus the reggeized spin-averaged differential cross section for the exchange of a given fermion trajectory can simply be written down as

$$
\begin{aligned}
\tau^2\frac{\mathrm{d}\sigma}{\mathrm{d}u} \sim \big(&[A(\sqrt{u})][B(\sqrt{u})]\tau^{2\alpha(\sqrt{u})}|\xi_\pm(\sqrt{u})|^2\\
&+[A(-\sqrt{u})][B(-\sqrt{u})]\tau^{2\alpha(-\sqrt{u})}|\xi_\pm(-\sqrt{u})|^2\big)
\end{aligned}
\tag{26}
$$

where the second term is the contribution of the opposite parity partner† and the square brackets are those of equation (25).

The s channel helicity form of the vertex traces to leading order have been calculated for different vertices and the 'residue functions' within the square brackets in equation (26) are listed in table 2.

Table 2. Vertex traces (s helicity form)

Vertex	Trace
$(0, \frac{1}{2}, j+\frac{1}{2})^+$	$\frac{1}{2m}\lvert g\rvert^2$
$(0, \frac{3}{2}, j+\frac{1}{2})^+$	$\frac{1}{2m'}\left(\left\lvert\frac{\sqrt{-u}}{2\sqrt{2}}g_2\right\rvert^2+\frac{1}{3}\left\lvert\frac{\mathrm{i}\sqrt{-u}}{2\sqrt{2}}g_2+\frac{\sqrt{2}}{m'}\{g_1+(\mu^2-m'^2-u)g_2\}\right\rvert^2\right)$
$(1, \frac{1}{2}, j+\frac{1}{2})^+$	$\frac{1}{2m}\left(\frac{1}{\mu'^2}\lvert g_1+(m-\sqrt{u})g_2+\frac{1}{2}(m^2-\mu'^2-u)g_3\rvert^2 + \left\lvert \mathrm{i}\sqrt{2}g_2+\frac{\sqrt{-u}}{2\sqrt{2}}g_3\right\rvert^2+\left\lvert\frac{\sqrt{-u}}{2\sqrt{2}}g_3\right\rvert^2\right)$
$(\gamma, \frac{1}{2}, j+\frac{1}{2})^+$	$\frac{1}{m}\{\lvert(m+2\sqrt{u})\tilde{g}_2+\tilde{g}_1\rvert^2+\lvert\sqrt{u}\tilde{g}_2\rvert^2\}$
$(\gamma, \frac{3}{2}, j+\frac{1}{2})^+$	$\frac{1}{2m'}\left(\left\lvert\frac{m'}{2}(\tilde{g}_2+\sqrt{u}\tilde{g}_4)\right\rvert^2+\left\lvert\frac{u}{2}\tilde{g}_4\right\rvert^2 +\frac{1}{3}\left\lvert\frac{2}{m'}\tilde{g}_1-\sqrt{u}\tilde{g}_2+2\tilde{g}_3-\frac{1}{2}(m'^2+2u)\tilde{g}_4\right\rvert^2 +\frac{1}{3}\left\lvert\frac{\sqrt{-u}}{2m'}\{\sqrt{u}\tilde{g}_2-4\tilde{g}_3+(2m'^2+u)\tilde{g}_4\}\right\rvert^2\right)$

Abnormal traces: $g(\sqrt{u}) \rightarrow f(\sqrt{u}), \quad \sqrt{u} \rightarrow -\sqrt{u}$

For example, the N_α exchange contribution to the cross section of backward photoproduction process, $\gamma\mathrm{p} \rightarrow \pi^+\mathrm{n}$, is

$$\begin{aligned}\tau^2\frac{\mathrm{d}\sigma}{\mathrm{d}u} \sim \frac{1}{2m}\lvert f(\sqrt{u})\rvert^2\left(\frac{1}{m}\{\lvert(m+2\sqrt{u})\tilde{g}_2(\sqrt{u})+\tilde{g}_1(\sqrt{u})\rvert^2+\lvert\sqrt{u}\tilde{g}_2(\sqrt{u})\rvert^2\}\right)\\ \times\tau^{2\alpha(\sqrt{u})}\lvert\xi(\sqrt{u})\rvert^2+\frac{1}{2m}\lvert f(-\sqrt{u})\rvert^2\\ \times\left(\frac{1}{m}\{\lvert(m-2\sqrt{u})\tilde{g}_2(-\sqrt{u})+\tilde{g}_1(-\sqrt{u})\rvert^2+\lvert-\sqrt{u}\tilde{g}_2(-\sqrt{u})\rvert^2\}\right)\\ \times\tau^{2\alpha(-\sqrt{u})}\lvert\xi(-\sqrt{u})\rvert^2. \qquad (27)\end{aligned}$$

In all the above calculations we have not used any symmetry restrictions to the vertex coupling functions except parity (and of course gauge invariance for photon vertices). Other restrictions such as G or C parity and statistics do not arise in our case. However, one could easily generalize the present technique to theories with higher symmetries, for example, U(6)⊗U(6), and the resulting relations among the residue functions could be used to reduce some of the free parameters of a model in checking against experiments. Work on this problem is in progress in this department.

† The residue functions obey the MacDowell symmetry relation: $\lvert g\sqrt{(u)}\rvert = \lvert f(-\sqrt{u})\rvert$.

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Appendix 1. Photon vertices

A1.1. Gauge invariance

The $\mathscr{M}$ function for processes involving one photon of momentum k, ($k^2 = 0$), can be written as (Gault *et al.* 1970, Jones and Scadron 1969 and Bardeen and Wu-ki Tung 1968)

$$\tilde{\mathscr{M}}_\nu = \sum_i \tilde{A}_i \tilde{\mathscr{K}}_\nu{}^i \tag{A1}$$

where the $\tilde{A}_i$ are invariant amplitudes free of kinematic singularities (KSF) and zeros (KZF) in s and u the $\tilde{\mathscr{K}}_\nu{}^i$ are linear combinations of covariants like Λ', Λ and $\bar{K}$, and $\tilde{\mathscr{M}}_\nu$ satisfies the gauge condition $\tilde{\mathscr{M}}_\nu k_\nu = \tilde{\mathscr{K}}_\nu k_\nu = 0$.

Gauge invariance is guaranteed if we write $\tilde{\mathscr{M}}_\nu$ as $\tilde{\mathscr{M}}_\nu = \mathscr{M}_{\nu'}\, \mathscr{G}_{\nu'\nu}$, where $\mathscr{G}_{\nu'\nu}$ is the gauge projection operator defined by (Jones and Scadron 1969, Bardeen and Wu-ki Tung 1968)

$$\mathscr{G}_{\nu'\nu} = g_{\nu'\nu} - \frac{k_{\nu'}\Lambda_\nu}{k.\Lambda} \tag{A.2}$$

which unfortunately introduces kinematic zeros into the invariant amplitudes (at $k.\Lambda = 0$). A way out of this situation is to regularize the amplitudes by first of all forming all possible less singular combinations and then multiplying by a factor of $(k.\Lambda)$ (Jones and Scadron 1969, Bardeen and Wu-ki Tung 1968).

In our case, the $\mathscr{M}$ function can be written as

$$\tilde{\mathscr{M}}_\nu = \mathscr{C}(\Lambda') : \mathscr{S}^J(\bar{K}) : \tilde{\mathscr{C}}_\nu(\Lambda) \tag{A.3}$$

where the photon vertex is gauge invariant, that is, $\tilde{\mathscr{C}}_\nu(\Lambda)k_\nu = 0$ with $\tilde{\mathscr{C}}_\nu(\Lambda) = \mathscr{C}_{\nu'}(\Lambda)\, \mathscr{G}_{\nu'\nu}$. We now consider the guage invariant forms of particular photon vertices.

A1.2. Photon–nucleon Regge vertex

For normal exchange, the gauge invariant vertex is simply

$$\tilde{\mathscr{C}}_\mu(\gamma, \tfrac{1}{2}, j+\tfrac{1}{2}) = (g_1 g_{\mu\alpha_1}' + g_2 \gamma_\mu' \Lambda_{\alpha_1})\Lambda_{\alpha_2} \ldots \Lambda_{\alpha_j} \tag{A.4}$$

where

$$g_{\mu\alpha}' = \mathscr{G}_{\mu\mu'} g_{\mu'\alpha} = g_{\mu\alpha} - \frac{\Lambda_\mu k_\alpha'}{k'.\Lambda} = g_{\mu\alpha} - \frac{\Lambda_\mu \Lambda_\alpha}{k'.\Lambda}$$

$$\gamma'_\mu = \mathscr{G}_{\mu\mu'}\gamma_{\mu'} = \gamma_\mu - \frac{\Lambda_\mu \gamma.k'}{k'.\Lambda'}$$

and

$$\Lambda_\mu' = \mathscr{G}_{\mu\mu'}\Lambda_{\mu'} = 0.$$

Since (Scadron 1968) $k'.\Lambda = \frac{1}{4}(m^2 - M^2)$ and $\gamma.k' \to (m-M)$†, the linear combinations

$$(M-m)g_{\mu\alpha}' + \gamma_\mu' \Lambda_\alpha = (M-m)g_{\mu\alpha} + \gamma_\mu \Lambda_\alpha \equiv \tilde{\mathscr{C}}_{\mu\alpha}{}^{(1)}$$

† We use the relations: $(\gamma.\bar{K}+M)\gamma.\bar{K} = (\gamma.\bar{K}+M)M$ and $(\gamma.p - m)u(p) = 0$ for equation (19) and the definition $\sigma_\pm = \sigma_1 \pm i\sigma_2$ for equation (20).

and

$$(M+m)\gamma_\mu'\Lambda_\alpha = (M+m)\gamma_\mu\Lambda_\alpha - 4\Lambda_\mu\Lambda_\alpha \equiv \tilde{\mathscr{C}}_{\mu\alpha}{}^{(2)}$$

are free of kinematic singularities.

The regularized couplings are thus given by

$$\tilde{\mathscr{C}}_\mu(\gamma, \tfrac{1}{2}, j+\tfrac{1}{2}) = (\tilde{g}_1\tilde{\mathscr{C}}_{\mu\alpha_1}{}^{(1)} + \tilde{g}_2\tilde{\mathscr{C}}_{\mu\alpha_1}{}^{(2)}\Lambda_{\alpha_2}\ldots\Lambda_{\alpha_j} \tag{A.5}$$

where $\tilde{g}$ are (KSF) in u.

A1.3. *Photon*–N* *Regge vertex*

Normal exchange leads to the covariants $g_{\beta_2\nu}'g_{\beta_1\mu}$, $\gamma_\nu'\Lambda_{\beta_2}'g_{\beta_1\mu}$, $g_{\mu\nu}'\Lambda_{\beta_1}'\Lambda_{\beta_2}'$ and $\gamma_\nu'\Lambda_\mu'\Lambda_{\beta_1}'\Lambda_{\beta_2}'$ where

$$g_{\beta_2\nu}' = g_{\beta_2\nu} - \frac{\Lambda_\nu'k_{\beta_2}}{k.\Lambda'} = g_{\beta_2\nu} - \frac{4\Lambda_\nu'\Lambda_{\beta_2}'}{m'^2-M^2}$$

$$g_{\mu\nu}' = g_{\mu\nu} - \frac{\Lambda_\nu'k_\mu}{k.\Lambda'} = g_{\mu\nu} - \frac{8\Lambda_\nu'\Lambda_\mu'}{m'^2-M^2}$$

and

$$\gamma_\nu' = \gamma_\nu - \frac{\Lambda_\nu'\gamma.k}{k.\Lambda'} = \gamma_\nu - \frac{4\Lambda_\nu'}{(m'+M)} \tag{A.6}$$

where the relations (Scadron 1968) $k.\Lambda' = \frac{1}{4}(m'^2-M^2)$ and $\gamma.k \to (m'-M)$ have been used.

Thus the linear combinations

$$(M-m')g_{\beta_2\nu}'g_{\beta_1\mu} + \gamma_\nu'\Lambda_{\beta_2}'g_{\beta_1\mu} = (M-m')g_{\beta_2\nu}g_{\beta_1\mu} + \gamma_\nu\Lambda_{\beta_2}'g_{\beta_1\mu} \equiv \tilde{\mathscr{C}}^{(1)}_{\mu\nu\beta_1\beta_2}$$

$$\begin{aligned}
\frac{(M-m')}{2}g_{\mu\nu}'\Lambda_{\beta_1}'\Lambda_{\beta_2}' + \gamma_\nu'\Lambda_\mu'\Lambda_{\beta_1}'\Lambda_{\beta_2}' &= \frac{(M-m')}{2}g_{\mu\nu}\Lambda_{\beta_1}'\Lambda_{\beta_2}' + \gamma_\nu\Lambda_\mu'\Lambda_{\beta_1}'\Lambda_{\beta_2}' \\
&\equiv \tilde{\mathscr{C}}^{(2)}_{\mu\nu\beta_1\beta_2'} \\
(M+m')\gamma_\nu'\Lambda_{\beta_2}'g_{\beta_1\mu} &= (M+m')\gamma_\nu g_{\beta_1\mu}\Lambda_{\beta_2}' - 4g_{\beta_1\mu}\Lambda_\nu'\Lambda_{\beta_2}' \\
&\equiv \tilde{\mathscr{C}}^{(3)}_{\mu\nu\beta_1\beta_2'}
\end{aligned}$$

and

$$\begin{aligned}
(M+m')\gamma_\nu'\Lambda_\mu'\Lambda_{\beta_1}'\Lambda_{\beta_2}' &= (M+m')\gamma_\nu\Lambda_\mu'\Lambda_{\beta_1}'\Lambda_{\beta_2}' - 4\Lambda_\mu'\Lambda_\nu'\Lambda_{\beta_1}'\Lambda_{\beta_2}' \\
&\equiv \tilde{\mathscr{C}}^{(4)}_{\mu\nu\beta_1\beta_2'}
\end{aligned}$$

are (KSF) in u and the final (KSF) vertex is

$$\tilde{\mathscr{C}}_{\mu\nu}(\gamma, \tfrac{3}{2}, j+\tfrac{1}{2}) = \sum_{i=1}^{4}\tilde{g}_i\tilde{\mathscr{C}}^{(i)}_{\mu\nu\beta_1\beta_2}\Lambda_{\beta_3}\ldots\Lambda_{\beta_j}. \tag{A.7}$$

Appendix 2. *s* channel helicity amplitudes

The s channel helicity amplitudes (Jacob and Wick 1959, Gault and Jones 1971) for the πN backward scattering can be written as (equation (12))

$$T_{\lambda';\lambda}{}^{J} \sim \bar{u}^{(\lambda')}(p')\mathscr{M}^J u^{(\lambda)}(p) \tag{B.1}$$

where we take $\boldsymbol{p}$ along $\hat{\mathbf{e}}_3$, $\boldsymbol{q}$ along $-\hat{\mathbf{e}}_3$, $\boldsymbol{p}'$ at an angle $\theta_s(\phi = 0)$ with respect to $\hat{\mathbf{e}}_3$ and $\boldsymbol{q}'$ at $(\pi - \theta_s)(\phi = \pi)$ with respect to $\hat{\mathbf{e}}_3$ (see figure 2).

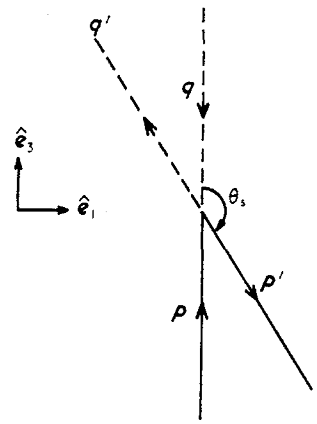

Figure 2.

The Dirac spinors are given by

$$u^{(\lambda)}(p) = \left(\frac{E+m}{2m}\right)^{1/2}\left(1+\frac{\mathrm{i}\gamma_5 p h}{E+m}\right)\phi^{(\lambda)}(\hat{\boldsymbol{p}})$$

$$\bar{u}^{(\lambda')}(p') = \left(\frac{E'+m}{2m}\right)^{1/2}\phi^{(\lambda')}(\hat{\boldsymbol{p}}')\left(1-\frac{\mathrm{i}\gamma_5 p' h'}{E'+m}\right) \qquad \text{(B.2)}$$

where

$$\phi^{(1/2)}(\hat{\boldsymbol{p}}) = \begin{pmatrix}1\\0\end{pmatrix} \qquad \phi^{(-1/2)}(\hat{\boldsymbol{p}}) = \begin{pmatrix}0\\1\end{pmatrix}$$

$$\phi^{(1/2)}(\hat{\boldsymbol{p}}') = \begin{pmatrix}\cos\frac{1}{2}\theta_s\\ \sin\frac{1}{2}\theta_s\end{pmatrix} \qquad \phi^{(-1/2)}(\hat{\boldsymbol{p}}') = \begin{pmatrix}-\sin\frac{1}{2}\theta_s\\ \cos\frac{1}{2}\theta_s\end{pmatrix}$$

and $p = |\boldsymbol{p}|$, $h = 2\lambda$.

In the Pauli representation, $\gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3$, $\boldsymbol{\gamma} = \gamma_0(\mathrm{i}\gamma_5)\boldsymbol{\sigma}$, and $\phi^+\gamma_5\phi = 0$, $\gamma_0\phi = \phi$.

In the case of the process $\pi\mathrm{N} \to \mathrm{N}\rho_\mu$, equation (12) becomes

$$T^J_{\lambda''\lambda';\lambda} \sim \bar{u}^{(\lambda')}(p')\mathscr{M}_\mu{}^J\epsilon_\mu{}'^{*(\lambda'')}(q')u^{(\lambda)}(p) \qquad \text{(B.3)}$$

where the polarization vector for spin 1 particle is given by

$$\epsilon_\mu{}'^{(0)}(q') = \frac{1}{\mu'}(q';\ \omega'\sin\theta_s,\ 0,\ -\omega'\cos\theta_s)$$

and

$$\epsilon_\mu{}'^{(\pm)}(q') = \pm\tfrac{1}{2}(0;\ \cos\theta_s,\ \mp\mathrm{i},\ \sin\theta_s).$$

For the process $\pi\mathrm{N} \to \mathrm{N}_\mu{}^*\pi$, the amplitude is

$$T^J_{\lambda';\lambda} = \bar{u}_\mu{}^{(\lambda')}(p')\mathscr{M}_\mu{}^J u^{(\lambda)}(p) \qquad \text{(B.4)}$$

with

$$u_\mu{}^{(\lambda)}(p) = \sum_{r,s}(1;\tfrac{1}{2},r,s|\tfrac{3}{2},\lambda)\epsilon_\mu{}^{(r)}(p)u^{(s)}(p)$$

where $(1, \frac{1}{2}, r, s|3/2, \lambda)$ are the Clebsch–Gordon coefficients and ϵ_μ and u are the usual spin 1 polarization vector and Dirac spinor, respectively (Scadron 1968).

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